Effect of Shock-Induced Vorticity on the Compressible Boundary Layer along a Flat Plate

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Along the conical surface or the flat surface of a blunt-nosed body in high-speed flight there exists a vortical layer induced by the curved shock It is the purpose of this paper to study the effect of the vortical layer on the boundary layer by the systematic procedure of matching the solution of the inner boundary layer to that of the outer layer In obtaining the next-order solution to the boundary layer, the difficulty lies in establishing the proper next-order boundary conditions or the matching conditions with the outer layer In the analysis of the present problem this difficulty is resolved by using the fact that, associated with vortical layer, there is a steep density gradient with density decreasing rapidly toward the wall In the matching region the density is very low; hence, it is permissible to neglect the product of density and gradient of normal velocity as compared to the product of normal velocity and density gradient With this approximation, the present paper shows that it is then possible to deduce proper matching conditions for the next-order inner solution from the governing equations of the next-order outer solution without actually solving them To illustrate this procedure, detailed analysis has been carried out for the simple model of compressible vortical layer over a flat plate to simulate the flow near the flat surface of a blunted wedge

Introduction

FOR a body with blunt nose in high-speed flight, there exists along the surface of the body a vortical layer induced by the curved shock Across the vortical layer there are steep velocity, density, and temperature gradients and a small pressure gradient The engineering significance of the velocity gradient for the characteristics of the boundary layer was first pointed out by Ferri and Libby [1] A simplified mathematical model of an incompressible external shear flow on the boundary layer along a flat plate was investigated by Li[2] and others The same incompressible model with large external shear was investigated by Ting [3]

Near the nose of the blunt body, i e , near the stagnation point, there is also a steep velocity gradient in the inviscid layer, but the density and temperature gradients are very small Furthermore, with the assumption of a cold wall, the displacement thickness is negligible The effects of external vorticity on the boundary-layer characteristic near the stagnation point, in absence of the effect of displacement thickness, have been investigated by Ferri et al ,[4] Cheng,[5] and others quoted therein

In studying the effect of the vortical layer on the compressible boundary layer along the surface of the blunt-nosed body where the effect of displacement thickness is no longer negligible, the real difficulty lies in how to establish the proper next-order boundary condition or the matching conditions with the outer vortical layer The standard procedure calls for the solution of the next-order outer solution due to the displacement thickness of the boundary layer Such a solution would require tedious numerical solution by characteristics and will not furnish analytical boundary conditions for the next-order analysis of the boundary layer In addition, the necessity of prescribing the conditions on the outside of the vortical layer, as in the forementioned incompressible cases, is also present It is the purpose of this paper to point out how this essential difficulty is eliminated by using the fact that, in the shock-induced vortical layer, there is a steep density gradient with density decreasing rapidly toward the wall [6] In the matching region the density is very low; hence, it is permissible to neglect the product of density and the gradient of normal component of velocity as compared to the product of the normal velocity and the density gradient With this approximation, it is then possible to deduce the proper matching conditions for the next-order inner solution from the governing equations of the next-order outer solution without actually solving them To illustrate the essential features of this procedure, the simple model of a flat plate is considered in the present paper Also, for the purpose of simplification, the usual assumptions that the Prandtl number is equal to unity and the viscosity in the boundary layer is proportional to its temperature are retained The application of this procedure to the problem of heat transfer at the conical surface of a blunt body with less restrictions on the Prandtl number and on the law of viscosity will be reported later

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Flat-Plate Problem

With the flat plate coinciding with the positive x axis, the inviscid parallel flow field is given by the prescribed velocity variations $u = U(y)$ and $v = 0$, and density variations $\rho_0(y)$ with constant pressure P and constant stagnation enthalpy H The velocity and density gradients are assumed to be positive, i e , $dU/dy > 0$, $d\rho_0/dy > 0$, to simulate a typical flow field of shock-induced vortical layer along the surface of a wedge with increasing entropy toward the plate (see Fig 1) Let the scale of the vortical layer be chosen as unity† and the scale of the boundary layer be denoted by δ It will be assumed that $\delta \ll 1$ so that the effect of the velocity gradient on the boundary layer is of the order of δ

Because of the displacement thickness of the boundary layer or the nonvanishing vertical component of the velocity, the effect of the boundary layer on the outer layer is also of the order δ

† Because of the absence of x dependence in the basic flow along the flat plate, there is no necessity to introduce a different x scale

In studying the next-order effect, it can be shown that the outer layer is still governed by the inviscid equations, whereas the inner layer is governed by the usual boundary-layer equations i e ,

$$(\bar{\rho}\bar{u})_x + (\bar{\rho}\bar{v})_y = 0 \tag{1}$$

$$\bar{\rho}\bar{u}\bar{u}_x + \bar{\rho}\bar{v}\bar{u}_y = -\bar{p}_x + (\partial/\partial y)\bar{\mu}(\partial\bar{u}/\partial y) \tag{2}$$

$$\bar{p}_y = 0 \tag{3}$$

$$\bar{\rho}\bar{u}(\bar{h})_x + \bar{\rho}\bar{v}(\bar{h})_y = (\partial/\partial y)\bar{\mu}(\partial\bar{h}/\partial y) \tag{4}$$

where barred quantities indicate those associated with the boundary layer In Eq (4) the usual assumption of $Pr = 1$ has been introduced for the sake of simplicity only

The physical quantities in the inner and outer layers are now expanded in terms of powers of δ as follows:

$$\begin{aligned} \bar{u} &= \bar{u}^{(0)} + \delta\bar{u}^{(1)} & \bar{v} &= \delta\bar{v}^{(0)} + \delta^2\bar{v}^{(1)} \\ \bar{\rho} &= \bar{\rho}^{(0)} + \delta\bar{\rho}^{(1)} & \bar{p} &= P + \delta\bar{p}^{(1)} \\ \bar{h}_s &= \bar{h}_s^{(0)} + \delta\bar{h}_s^{(1)} & u &= U + \delta u^{(1)} \\ v &= \delta v^{(1)} & p &= \bar{P} + \delta p^{(1)} \\ \rho &= \rho^{(0)} + \delta\rho^{(1)} & h_s &= H + O(\delta^2) \end{aligned} \tag{5}$$

The standard match conditions are (see, for example, Refs 7–9)

$$\lim_{Y\to\infty} \bar{\Lambda}^{(0)} = \Lambda^{(0)}(y = 0) \tag{6a}$$

$$\lim_{Y\to\infty}\left[\bar{\Lambda}^{(1)} - Y\left(\frac{d\Lambda^{(0)}}{dy}\right)_{y=0}\right] = \Lambda^{(1)}(y = 0) \tag{6b}$$

$$\lim_{Y\to\infty}\left[\bar{\Lambda}^{(2)} - \frac{Y^2}{2}\left(\frac{d\Lambda^{(0)}}{dy}\right)_{y=0} - Y\left(\frac{d\Lambda^{(1)}}{dy}\right)_{y=0}\right] = \Lambda^{(1)}(y = 0) \tag{6c}$$

$$v^{(1)}(y = 0) = \bar{v}^{(0)}(y \to \infty), \text{ etc} \tag{7}$$

where $Y = y/\delta$ and Λ stands for u, ρ, p, or h_s

Zero-Order Boundary-Layer Solution

With the standard external boundary condition of Eq (8) and a constant wall temperature of T_w, the zero-order solution is identical with the standard boundary-layer solution (see, for example Ref 10) The relevant results are listed in the following:

$$\bar{u}^{(0)} = U_e f'(\eta) \tag{8a}$$

$$\bar{h}^{(0)} = h_w + (H - h_w)f'(\eta) \tag{8b}$$

$$g(\eta) = \frac{\bar{h}^{(0)}}{h} = \frac{\rho}{\bar{\rho}^{(0)}} = \frac{h_w}{h}(1 - f') + f' + \frac{\gamma - 1}{2} M_e^2 f'(1 - f') \tag{8c}$$

$$G(\eta) = \int_0^\eta \frac{\rho_e}{\bar{\rho}^{(0)}}\, d\eta = \frac{h_v}{h}(\eta - f) + f + \frac{\gamma - 1}{2} M^2[f - ff' - f'' + f''(0)] \tag{8d}$$

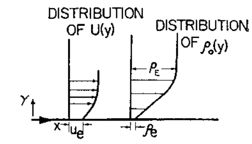
$$\bar{v}^{(0)} = (\nu U/2x)^{1/2}(Gf' - gf) \tag{8e}$$

where quantities with the subscripts "e" denote those of the zero-order inviscid flow at the plate [e g , $U = U(y = 0)$, $\rho = \rho^{(0)}(y = 0)$, etc], M denotes the Mach number, h the static enthalpy, and ν the kinematic viscosity η is the transformed variable defined by the equation

$$\eta = \int_0^y \frac{\rho^{(0)}}{\rho_e} \frac{dy}{(2x\nu/U)^{1/2}} \tag{9}$$

and f is the standard Blasius function of η fulfilling the

Fig 1 Simple model of a constant pressure compressible vortical layer along a flat plate



differential equation $f''' + ff'' = 0$ with the $f(0) = f'(0) = 0$ and $f''(0) = 0\ 4696$

For large values of η, the asymptotic behavior of the functions f, g G, and $\bar{v}^{(0)}$ are as follows:

$$f \to \eta - \beta + 0(e^{-\eta^2}) \tag{10a}$$

$$g \to 1 + 0(e^{-\eta^2}) \tag{10b}$$

$$G \to \eta - \beta + \varkappa \tag{10c}$$

$$v^{(0)} \to \varkappa U/(2x)^{1/2} \tag{10d}$$

where

$$\beta = 1\ 217 \qquad \varkappa = \frac{h_w\beta}{h} + \frac{\gamma - 1}{2} M^2 f'(0)$$

and δ is defined as $(\nu/U_e)^{1/2}$

Matching Conditions for the Next-Order Boundary-Layer Solution

With Eq (7) as the boundary condition, the next-order outer flow is governed by the linearized inviscid equations, namely,

$$\rho_0 u_x^{(1)} + \rho_x^{(1)} U + \rho_0 v_y^{(1)} + (\rho_0)_y v^{(1)} = 0 \tag{11a}$$

$$\rho_0 U u_x^{(1)} + \rho_0 v^{(1)} U_y = -p_x^{(1)} \tag{11b}$$

$$\rho_0 U v_x^{(1)} = -p_y^{(1)} \tag{11c}$$

$$h^{(1)} = Uu^{(1)} + h^{(1)} = 0 \tag{11d}$$

where $h^{(1)}$ is related to $\rho^{(1)}$ and $p^{(1)}$ by the equation of state

Equation (11c) at $y = 0$ defines $p_y^{(1)}$, which term will be used for the behavior of the second-order boundary-layer solution $\bar{p}^{(2)}$ as $Y \to \infty$ by means of the matching condition (6c) Equations (11a, 11b, and 11d) at $y = 0$ would provide the values of first-order solutions $\rho^{(1)}$, $u^{(1)}$, and $h^{(1)}$ or $p^{(1)}$ at $y = 0$ required by the boundary conditions of Eq (6b), provided that $v_y^{(1)}(y = 0)$ in Eq (11a) is known or the term $\rho_0 v^{(1)}$ at $y = 0$ can be neglected In general, $v_y^{(1)}$ at $y = 0$ is known only after the solution of the differential equations (11), together with the proper boundary-layer conditions along the inner and outer edges of the vortical layer In general, it can be solved only numerically The question of what is the proper boundary condition on the outer edge of the vortical layer further complicates the problem

For the shock-induced vortical layer, there is a large entropy increase toward the wall; hence the density at the wall ρ is much smaller than the density at the outer edge of the layer ρ_E (see sketch in Fig 1) With the fact that $\rho_e/\rho_E \gg 1$, it is now possible to show that the term $\rho_0 v_y^{(1)}$ is much smaller than the term $(\rho_0)v^{(1)}$ in Eq (11a) at $y = 0$ by the following order-of-magnitude analysis In this paper, the thickness of the vortical layer has been chosen as unity This implies that, in the vortical layer, $v^{(1)}$ and $v_y^{(1)}$ are of the same order of magnitude and that $\rho_y^{(0)}$ is of the order of $(\rho_E - \rho_e)$ It becomes clear that

$$v^{(1)}\rho_y^{(0)} = O(v_y^{(1)})(\rho_E - \rho) = O(v_y^{(1)}\rho)(\rho_E/\rho) \gg v_y^{(1)}\rho = O(\rho_0 v_y^{(1)}) \text{ at } y = 0$$

Omission of the term $\rho_0 v_y^{(1)}$ in Eq (11a) at $y = 0$ implies an error of the order of ρ/ρ_E With this omission, the first-order solution obtained in this paper will reduce the error of the zero-order solution to the order of $\delta(\rho_e/\rho_E)$ or δ^2, which-

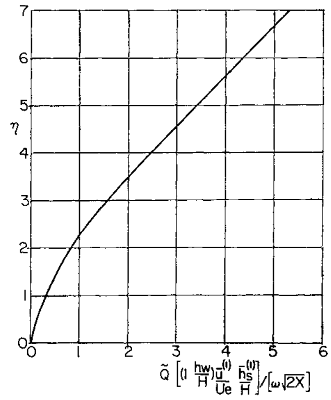


Fig 2 Distribution of the linear combination of velocity and stagnation enthalpy ($M_e = 2$, $h_w/h = 0\ 1$)

ever is greater With the term of $\rho_0 v_y^{(1)}$ neglected in Eq (11a) at $y = 0$, Eqs (11a, 11b, and 11d) at $y = 0$ yield

$$\frac{u_x^{(1)}}{U} = -\frac{M_e^2 p_x^{(1)}}{\rho U_e^2} = -\frac{v^{(1)}\omega M^2}{U(M_e^2 - 1)} = -\frac{\varkappa\omega M_e^2}{(2x)^{1/2}(M_e^2 - 1)} \quad (12a)$$

$$\frac{\rho_x^{(1)}}{\rho_e} = -\frac{\lambda v^{(1)}}{U} - \frac{u_x^{(1)}}{U} = -\frac{M_e^2[M^2(\gamma - 1) - \gamma]\omega\varkappa}{(M^2 - 1)(2x)^{1/2}} \quad (12b)$$

where $\omega = (dU/dy)/U$ and $\lambda = (d\rho_0/dy)/\rho_e = (\gamma - 1)M^2\omega$

Integration of Eqs (12) with respect to x along $y = 0$ yields

$$\frac{u^{(1)}}{U} = -\frac{M^2 p^{(1)}}{\rho U^2} = -\frac{\varkappa\omega M_e^2(2x)^{1/2}}{M^2 - 1} \quad (13a)$$

$$\frac{\rho^{(1)}}{\rho} = -\frac{M_e^2[M^2(\gamma - 1) - \gamma]\omega\varkappa(2x)^{1/2}}{(M^2 - 1)} \quad (13b)$$

The constants of integration are set equal to zero either on the assumption that $u^{(1)}$, $p^{(1)}$, and $\rho^{(1)}$ vanish at $x = 0$ or according to the argument that, since the thickness of the vortical layer is chosen as the length scale, the nondimensional distance x along the plate at sufficient distance from the leading edge is much greater than unity, and the constant of integration can be ignored as compared to $(x)^{1/2}$

Equations (6b, 13a, and 13b) produce the necessary boundary conditions at $Y \to \infty$ or $\eta \to \infty$ for the next-order boundary-layer solution, namely,

$$\frac{\bar{u}^{(1)}}{U_e} \to \omega\frac{y}{\delta} + \frac{u^{(1)}(y = 0)}{U} = \omega(2x)^{1/2}\left[\eta - \beta - \frac{\varkappa}{M_e^2 - 1}\right] \quad (14a)$$

$$\frac{\bar{\rho}^{(1)}}{\rho} + \frac{\bar{u}^{(1)}}{u} \to \omega(2x)^{1/2}\{[(\nu - 1)M_e^2 + 1](\eta - \beta) + \varkappa\} \quad (14b)$$

$$\bar{h}^{(1)} \to 0 \quad (14c)$$

Next-Order Boundary-Layer Solution

The next-order continuity equation is fulfilled by the introduction of the stream function $\psi^{(1)}(x\ y)$ such that

$$\bar{\rho}^{(0)}\bar{u}^{(1)} + \bar{\rho}^{(1)}\bar{u}^{(0)} = \psi_y^{(1)} \quad (15a)$$

$$\bar{\rho}^{(0)}\bar{v}^{(1)} + \bar{\rho}^{(1)}\bar{v}^{(0)} = -\psi_x^{(1)} \quad (15b)$$

Because of the presence of the first-order pressure gradient $p^{(1)}(x,0)$, the first-order stagnation enthalpy $\bar{h}_s^{(1)}$ is no longer linearly related with the velocity $\bar{u}^{(1)}$ It is necessary to solve both the momentum and the energy equations simultaneously, because of the appearance of $\bar{u}^{(1)}$ and $\bar{\mu}^{(1)}$ or $\bar{h}^{(1)}$ in both equations They can be uncoupled if a new variable defined by the linear combination of h_s and u is introduced:

$$Q = [\bar{h}^{(0)} + \delta\bar{h}^{(1)}] - [h_w + (H - h_w)(\bar{u}^{(0)} + \delta\bar{u}^{(1)})U] = \delta\{\bar{h}_s^{(1)} - [(H - h_w)/U]\bar{u}^{(1)}\} \quad (16)$$

Since Q is of the order of δ, an uncoupled equation for Q is obtained by a linear combination of the momentum and energy equations The result is

$$\bar{\rho}^{(0)}\bar{u}^{(0)}Q_x + \bar{\rho}^{(0)}\bar{v}^{(0)}Q_y = \left(\frac{H - h_w}{U}\right)p_x^{(1)} + \frac{\partial}{\partial y}\bar{\mu}^{(0)}\frac{\partial Q}{\partial y} \quad (17)$$

A similar solution, $Q = \delta(2x)^{1/2}H\tilde{Q}\omega(\eta)$, is now introduced and Eq (17) becomes

$$\tilde{Q}'' + f\tilde{Q}' - f\tilde{Q} = -\left(1 - \frac{h_w}{H}\right)\frac{\varkappa}{(M_e^2 - 1)}g(\eta) \quad (18)$$

The boundary conditions are

$$\tilde{Q} = 0 \text{ at } \eta = 0$$
$$\tilde{Q} \to -\left(1 - \frac{h_w}{H}\right)\left(\eta - \beta - \frac{\varkappa}{M_e^2 - 1}\right) \text{ as } \eta \to \infty \quad (19)$$

Two numerical solutions $\tilde{Q}_0(\eta)$ and $\tilde{Q}_1(\eta)$ of Eq (18) will be obtained with the initial data $\tilde{Q}_0(0) = \tilde{Q}_0'(0) = 0$ and $\tilde{Q}_1(0) = 0$, $\tilde{Q}_1'(0) = 1$, respectively The solution $\tilde{Q}$ of Eqs. (18) and (19) is then expressed as a linear combination of $\tilde{Q}_0$ and $\tilde{Q}_1$, i e ,

$$\tilde{Q}(\eta) = a_0\tilde{Q}_0 + a_1\tilde{Q}_1 \quad (20)$$

The constants a_0 and a_1 are determined by the two following algebraic equations:

$$a_0 + a_1 = 1$$
$$a_0\tilde{Q}_0'(\infty) + a_1\tilde{Q}_1'(\infty) = -1 + (h_w/H) \quad (21)$$

Consequently, $\tilde{Q}$ will approach $-[1 - (h_w/H)](\eta - \tilde{\beta})$ as $\eta \to \infty$ and the constant $\tilde{\beta}$ should be equal to $\beta + \varkappa(M^2 - 1)$ and hence can serve as a check to the solution

Since the density integral

$$\int_0^y \frac{\bar{\rho}^{(0)}}{\rho}\,dy$$

in the definition of η in the zero-order solution is $\bar{\rho}^{(0)}$ and not $\bar{\rho}^{(0)} + \delta\bar{\rho}^{(1)}$, the first-order compressible boundary-layer equations will not be transformed into the equivalent incompressible equations in the variables x, Y or x, η By introducing the similar solutions

$$\psi^{(1)} = \omega\rho_e(\nu_e U)^{1/2}(2x)\tilde{\psi}(\eta) \quad (22a)$$

$$\bar{u}^{(1)} = U\,\omega(2x)^{1/2}V(\eta) \quad (22b)$$

$$\rho^{(1)} = \rho_e\omega(2x)^{1/2}R(\eta) \quad (22c)$$

the continuity equation or the definition of the stream func-

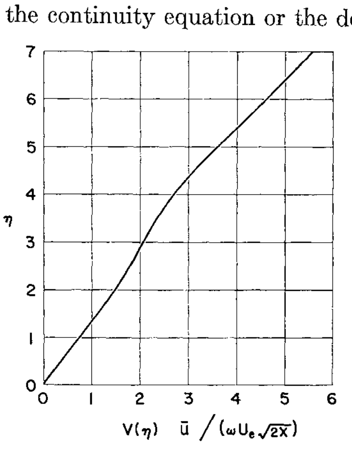


Fig 3 First-order velocity distribution ($M = 2$, $h_w/h = 0\ 1$)

tion $\tilde{\psi}^{(1)}$ yields

$$\tilde{\psi}' = Rf'g + V \quad (23)$$

The density function R is related to V and $\tilde{\psi}$ by the enthalpy function $\tilde{Q}$ as follows:

$$R = \frac{1}{g^2}\left[\frac{\gamma\varkappa M^2 g}{M^2 - 1} - \left(1 + \frac{\gamma - 1}{2} M^2\right)(Q + V) + (\gamma - 1)M^2 f\, V + \frac{h_w V}{h}\right] \quad (24)$$

The first-order x component of equation of motion is

$$\bar{u}^{(0)}\bar{\rho}^{(0)}\bar{u}_x^{(1)} + \bar{\rho}^{(0)}\bar{v}^{(0)}\bar{u}_y^{(1)} + \psi_y^{(1)}\bar{u}_x^{(0)} - \psi_x^{(1)}\bar{u}_y^{(0)} = -p_x^{(1)} + \frac{\partial}{\partial y}\bar{\mu}^{(0)}\frac{\partial \bar{u}^{(1)}}{\partial y} + \frac{\partial}{\partial y}\bar{\mu}^{(1)}\frac{\partial \bar{u}^{(0)}}{\partial y}$$

which, after the substitution of the similar solutions, becomes

$$V'' + fV' - f'V + 2\tilde{\psi}f'' + Rgff'' - (Rg)'f'' = \frac{\varkappa(g + \gamma M_e^2 ff'')}{M_e^2 - 1} \quad (25)$$

Equations (23) and (25) are the two differential equations for V and $\tilde{\psi}$ The boundary conditions are

$$\tilde{\psi} = 0 \qquad V = 0 \text{ at } \eta = 0 \quad (26a)$$

$$V \to \eta - \beta - [\varkappa/(M^2 - 1)] \text{ as } \eta \to \infty \quad (26b)$$

Similar to the determination of $\tilde{Q}$, two sets of numerical solutions, $\tilde{\psi}_0$, V_0 and $\tilde{\psi}_1$, V_1, to Eqs (23) and (25) are obtained with the initial data $\tilde{\psi}(0) = V(0) = V'(0) = 0$ and $\tilde{\psi}(0) = V(0) = 0$, $V'(0) = 1$, respectively The solutions of Eqs (23) and (25) fulfilling the boundary condition of Eqs (26) are obtained as follows:

$$\tilde{\psi} = b_0\tilde{\psi}_0 + b_1\tilde{\psi}_1 \quad (27a)$$

$$V = b_0V_0 + b_1V_1 \quad (27b)$$

with the constants b_0 and b_1 determined by the equations

$$b_0 + b_1 = 1 \qquad b_0V_0'(\infty) + b_1V_1'(\infty) = 1 \quad (28)$$

Again the constant $-\{\beta + [\varkappa/(M^2 - 1)]\}$ in Eq (26b) serves as a check to the accuracy of the solution

Numerical Example

The similar solutions $V(\eta)$ and $\tilde{Q}(\eta)$ depend on two parameters, M and h_w/h_e The vorticity parameter ω enters as a scale factor in the definition of the first-order physical quantities $Q(\eta,x)$ and $\bar{u}^{(1)}(\eta,x)$

The correction factors for shearing stress and heat-transfer rate at the wall due to the presence of the first-order terms are given by the following expressions:

$$\lambda_\tau = \frac{\tau_w^{(0)} + \tau_w^{(1)}\delta}{\tau^{(0)}} = 1 + C_\tau\omega\left(\frac{2\nu_e x}{U}\right)^{1/2} = 1 + C_\tau\left(\frac{2\nu x}{U}\right)^{1/2}\left(\frac{1}{U}\frac{dU}{dy}\right)$$

$$\lambda_{\dot{q}} = \frac{\dot{q}_w^{(0)} + \delta\dot{q}_w^{(1)}}{\dot{q}_w^{(0)}} = 1 + C_{\dot{q}}\left(\frac{2\nu x}{U}\right)^{1/2}\left(\frac{1}{U}\frac{dU_e}{dy}\right)$$

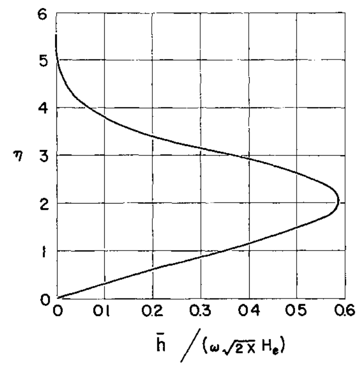


Fig 4 First-order stagnation enthalpy distribution ($M = 2$, $h_w/h = 0\ 1$)

where

$$C_\tau = V'(0)/f''(0)$$

$$C_{\dot{q}} = \frac{\tilde{Q}'(0) + [1 - (h_w/H)]V'(0)}{[1 - (h_w/H)]f''(0)}$$

C_τ and $C_{\dot{q}}$ are functions of M_e and h_w/h because $V'(0)$ and $\tilde{Q}'(0)$ are functions of these two parameters also

For the case $M = 2$ and $h/h_w = 0\ 1$, the solutions $\tilde{Q}(\eta)$ $V(\eta)$ are obtained by the method outlined in the preceding section Figures 2–4 show the curves of $\tilde{Q}(\eta)$, $V(\eta)$, and $\bar{h}^{(1)}/[\omega H(2x)^{1/2}]$, respectively With $V'(0) = 0\ 865$ and $\tilde{Q}''(0) = -0\ 297$, the correction factors for shearing stress and heat-transfer rate at the wall are

$$\lambda_\tau = 1 + 1\ 842\left(\frac{2\nu_e x}{U}\right)^{1/2}\left(\frac{1}{U}\frac{dU_e}{dy}\right)$$

$$\lambda_{\dot{q}} = 1 + 1\ 172\left(\frac{2\nu_e x}{U}\right)^{1/2}\left(\frac{1}{U_e}\frac{dU_e}{dy}\right)$$

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